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Designing with Plastics

Sample Chapter 5:
Calculations for Structures under Mechanical Load –
Examples of Geometrically Simple Structural Parts
under Static Loads

5　Calculations for Structures under Mechanical Load – Examples of Geometrically Simple Structural Parts under Static Loads

5.1　Specific Materials and Processing Problems

The mechanical properties of polymeric materials, especially those of thermoplastics, depend to a much greater extent on temperature, time, and on the magnitude and nature of an applied load than those of metals. In addition, many environmental effects, such as UV radiation or exposure to certain chemicals, play a significant role in aging and related changes in properties. This can be difficult to quantify in strength-related calculations. The conditions used in processing (*e.g.*, injection molding process) can have an effect on the properties of the finished product.

Because the strength of most polymeric materials is an order of magnitude less than that of metals, components made from these materials may be highly stressed even under relatively low loads. On the other hand, a component made from a polymeric material is more likely to be rendered unusable by a high degree of deformation than by catastrophic failure due to fracture (in the case of ductile polymeric materials). The modulus of elasticity of these materials differs by as much as two orders of magnitude. The complexity of deformation behavior, however, leads to the expectation that deformation can only be calculated precisely with the aid of computer support and that the empirical determination of physical properties for this purpose will be very costly. There are, however, a limited number of design tasks for which such extensive design work is necessary. Not all designers have the extensive knowledge required to optimize structural elements with the aid of FEM. This task is best performed by an engineer who specializes in computer aided finite element structural design. However, there are a large number of applications where routine structural design calculations can be done by the average designer to help determine optimum part geometry. These are carried out using justifiable simplifications [5.2]. Knowledge of some specific properties of polymers is needed for this purpose.

5.1.1　Deformation Behavior under Uniaxial Dynamic Tensile Stress

The deformation of polymeric materials under the action of an external force, can be described by three components, which to some extent are superimposed, but at the same time predominate in certain ranges of deformation (see also Section 4.1).

Linear elastic, spontaneously reversible deformation is restricted in most polymeric materials to a total strain range of less than one tenth of one percent. This marks the end of the range of applicability of Hooke's law of elasticity for many polymeric materials. Most strains observed

in practice exceed this range. In many unreinforced thermoplastics, it is not even possible to demonstrate (by way of stress-strain experiments) the existence of a range in which Hooke's law is obeyed.

As strain increases, visco-elastic deformation processes appear. These are said to fall into the *linear visco-elastic range* when the laws of linear visco-elasticity apply. This range is characterized in that two strains ε_1 and ε_2 may be added when the corresponding stresses σ_1 and σ_2 are superposed (Boltzmann's law of superposition).

$$
\begin{array}{ccc}
\sigma_1 & \Rightarrow & \varepsilon_1 \\
\underline{\sigma_2} & \Rightarrow & \underline{\varepsilon_2} \\
\sigma_1 + \sigma_2 & = & \varepsilon_1 + \varepsilon_2
\end{array}
\tag{5.1}
$$

In isochronous stress-strain diagrams, this range is marked by the end of the straight-line rise of the isochrones. This range also ends for most polymeric materials at stresses which cause strains of 0.5 to 1%. As strain increases further, the relationship of stress to strain no longer depends only on time (and of course on temperature), bur rather on the applied stress itself. The deformation processes become increasingly nonlinear and also irreversibly (or partially irreversibly) viscous.

These deformation phenomena found at the macroscopic level are characterized by the molecular deformation and damage mechanisms occurring in the material (see also Chapter 4).

In some applications, thermoplastic polymeric materials are subjected to loads which take them into the nonlinear stress-strain range. As a result, calculations based on the laws of elasticity inevitably yield results which diverge to a greater or lesser extent from actual behavior. As long as the stresses are at least close to the linear viscoelastic range, the formulae of the theory of elasticity afford a satisfactory degree of accuracy. Therefore, their use is certainly justifiable and may indeed be essential, based on time available and on economic grounds.

In addition, the calculation of structural elements made from polymeric materials is rendered more difficult and uncertain due to the fact that the condition of a homogeneous, isotropic continuum is not fulfilled. Here, the different types of anisotropy should be taken into account whenever possible (*i.e.*, when anisotropic data is available). The causes of these may lie in the material itself (reinforcement by unidimensional fibers, for instance) or be due to processing. Examples of the latter include residual molecular alignments and internal stresses imposed during the molding operation (see also Chapters 2 and 7).

Internal stresses can be caused by impediments to shrinkage in macroscopic domains. A number of primary causes of internal stress may be distinguished. Cooling internal stresses are the result of different rates of cooling over the cross section of the molding. Holding-pressure related internal stresses are due to the holding pressure acting in the interior of the molding in the injection molding process when the outer contours have already solidified. Embedding internal stresses arise from shrinkage impeded by the shape of the molding due, for example, to metal inserts or due to the constraints imposed by the shape of the mold itself. Secondary internal stresses are also known. These include structural internal stresses brought about by curing reactions (in thermosets) or crystallization (in semi-crystalline thermoplastics). Embedding internal stresses can be produced, for example, by the incorporation of fillers.

Cooling internal stresses give rise to compressive stresses on the surface of the molding. These compressive internal stresses can have a positive effect in the event of tensile loads in the outer zone (*e.g.*, during bending). On the other hand, excessive holding pressure causes compressive internal stresses in the interior of the molding and tensile internal stresses in the outer zone. These are superimposed on the cooling internal stresses so that when the holding pressure is high enough, tensile internal stresses may also be apparent on the surface of the molding. These internal stress states, which are also very difficult to describe quantitatively, represent a further uncertainty factor in calculations of strength.

5.2   Determination of Strength

5.2.1   Basic Procedure for Structural Part Design

The basic procedure for structural part design is given in Figure 5.1. An analysis of stress provides information about the magnitude and nature of the stresses at work in the theoretical cross section in question. Multiaxial states of stress are transformed using suitable failure criteria into a uniaxial reference stress having the same effect which is compared with the permissible level of stress. The latter is obtained from a characteristic property value specific to the material being considered for the application (*e.g.*, tensile yield strength etc.). This property value is further reduced by an appropriate safety factor and any applicable reduction factors.

According to this analysis, the fundamental equation for determining strength may be written down as:

$$\sigma_{v\,max} \leq \sigma_{perm} = \frac{K}{S \cdot A} \tag{5.2}$$

where

$\sigma_{v\,max}$ = maximum stress occurring in section being analyzed
K = characteristic strength property of the material
S = safety factor
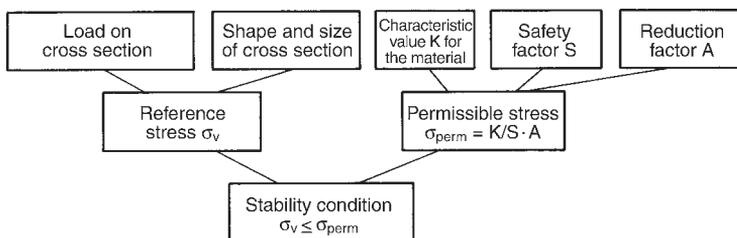A = material-specific reducing factor

Figure 5.1 Basic procedure for structural part design [5.1]

5.2.1.1   Characteristic Strength

A number of different (material specific) strength parameters can be used for structural design, depending on the specific material behavior.

Figure 5.2 shows the most important failure characteristics.

Materials exhibiting largely brittle failure (see Figure 5.2a, corresponding to type A in Table 4.1) or having a distinct yield point (see Figure 5.2b, corresponding to types B and C in Table 4.1) have a clear-cut characteristic value that can be used as a "failure stress". This is more difficult to define when there are no such prominent features in the stress-strain behavior, as is the case for many thermoplastics, especially at elevated temperatures. In these cases, a stress is proposed that causes a deformation or strain of a certain magnitude (0.5% offset has proved to be useful) in the nonlinear range (see Figure 5.2c, corresponding to type D in Table 4.1). In amorphous (transparent) polymeric materials, craze zones form when a certain damage stress is exceeded. Although in the initial stages these craze zones (see also Figure 4.2) are still capable of bearing loads, macroscopic cracks form in the course of further strain. This craze limit stress (onset of crazing) can be used as the failure value if available (see Figure 5.2d). Knit or weld line strength may be the limiting factor for structural parts containing weld lines (especially for reinforced thermoplastics).

In the case of sustained static loads, the creep strength is selected as the failure value (see Figure 5.3a) or the value σ^* in Section 4.2.1.

Under dynamic loads, the characteristic failure value is correspondingly obtained from the Wöhler curve or the Smith diagram (see Figures 5.3b and c). Alternatively, a limiting value as shown in Table 4.3 or a load-cycle-dependent rigidity value (see Figure 4.20) is introduced.
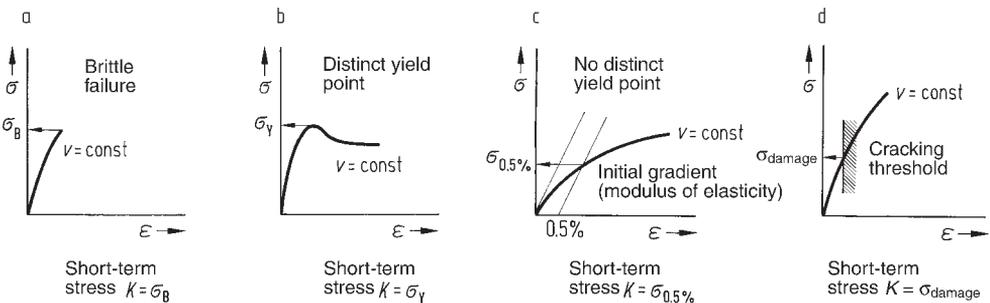

Figure 5.2   Some common short-term stress values that may be suitable as "failure stress" values [5.1]
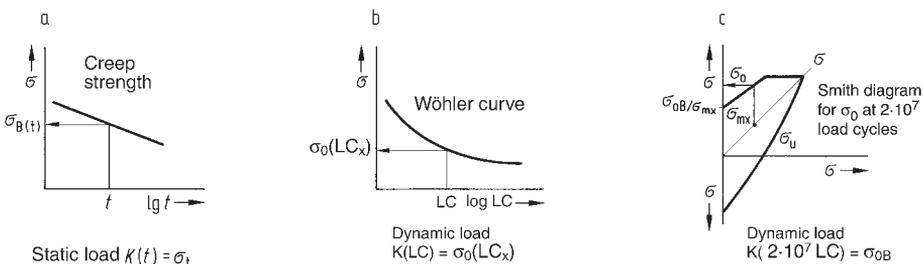

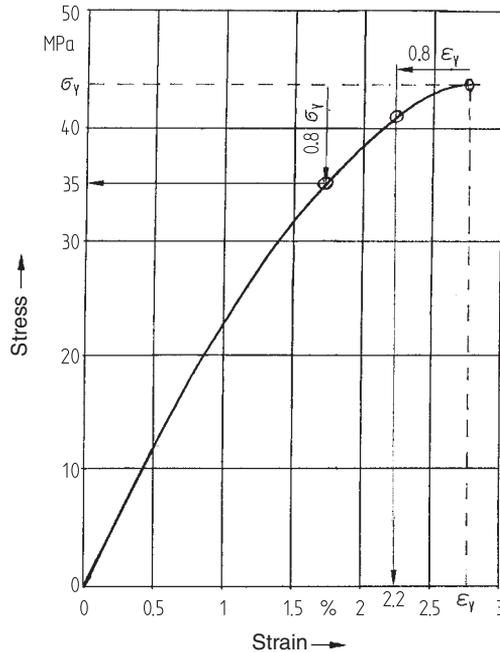**Figure 5.3**   Common dimensioning characteristics for long-term loads

Figure 5.4   A percentage safety interval in a strain-based analysis affords a permissible value different from that in a stress-based analysis for materials that exhibit non-linear stress-strain behavior

Of course, in determining characteristic failure values, the high temperature-dependence of thermoplastics must be taken into account. Here, heat may arise as an external effect or be generated by friction or damping. This situation generates problems in determining and using characteristic values for dynamic failure. This is especially true if long-term aging is a significant factor.

In many cases (*e.g.*, in snap-fit connections), strain-based structural design is more appropriate than stress-based designs. Design calculations are done on the basis of a limiting strain that must not be exceeded. It should be noted that in cases of nonlinear deformation behavior, there is a difference between operating, for example, at 80% of the yield *strain* and operating at 80% of the yield *stress*. Different values are obtained as depicted in Figure 5.4.

In [4.18] Oberbach proposes that permissible stress values be determined using a type of nomograph. The nomograph makes a distinction between short-term, long-term, and dynamic stresses; short-term stress being subdivided into single and multiple loading. A distinction is also made between ductile semi-crystalline, brittle amorphous, and glass-fiber reinforced thermoplastics. The procedure is illustrated in Figure 5.5.

The nature of the load and the group into which the material falls lead to an A-factor on the right ordinate of the left-hand part of the figure (in the example, single loading/glass-fiber reinforced thermoplastic = 0.68). The breaking load (in the example σ_B = 144 MPa) is multiplied by this factor to obtain the design stress at room temperature. The ordinate of the stress-strain diagram on the right-hand part of the figure is divided into fractions of the
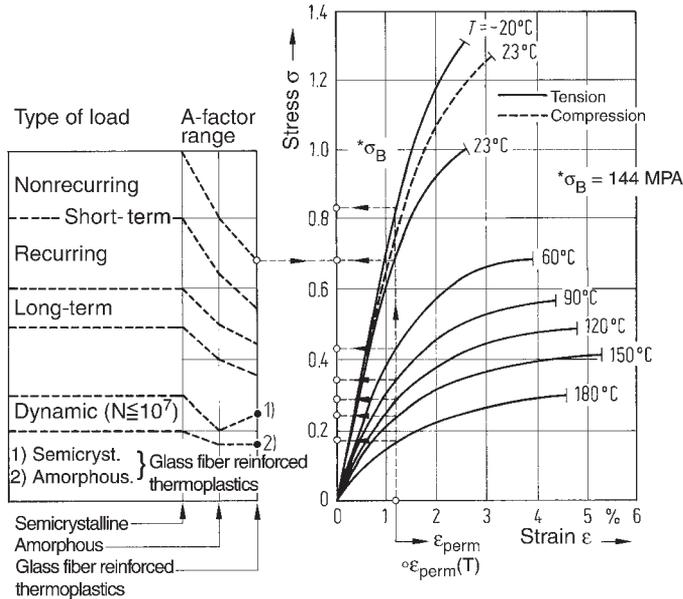
Figure 5.5   Estimation of permissible stresses and strains for a glass-fiber reinforced PBT (30% by weight)

breaking load at 23 °C. From the isotherms a permissible strain value can be determined at the requisite temperature for the design stress determined in this way (in the example ε_{perm} = 1.2%). While keeping this strain value constant, the A-factor at any other temperature can now be determined at the point of intersection with the corresponding isotherm. This allows the calculation of the permissible stress at this temperature. The same procedure may be used to determine the effect of time for isochronous stress-strain curves.

5.2.1.2   Safety Factors

The magnitude of the safety factors used in structural design of load bearing plastic parts depends on a number of variables, including the various uncertainties in calculating and determining the characteristic values of a material. Such uncertainties often arise, for example, when a load is first applied and are further complicated by simplified assumptions concerning geometry, stress, or previous processing. Secondly, the magnitude of safety factors depends on the seriousness of the damage which would occur were the structural part to fail. It is also influenced by the specific characteristic failure value against which the theoretical calculation is intended to provide a safeguard. It may be the designer who determines the magnitude of the safety factor in some applications. In some cases, agencies also prescribe minimum levels of safety for load-bearing structural parts.

The designer is responsible for the first group of safety factors identified above. He or she must estimate the effect of the simplifications made and the unquantifiable processing operations and rate them according to importance. For the second category of safety factors, the following guide values apply, provided they have not been previously described or agreed differently.

$S_{min} \geq 2$ for calculations safeguarding against fracture

$S_{min} \geq 3$ for calculations safeguarding against bending and buckling

$S_{min} \geq 1.2$ for calculations safeguarding against fracture stresses due to cracking with the additional condition that $S_{min} \geq 2$ for strength

$S_{min} = 1.0$ for calculations of $S_{0.5\%}$, with the additional condition that $S_{min} \geq 2$ for strength.

5.2.1.3   Reducing Factors

Other uncertainties attributable to the lack of physical properties data under special conditions should not be considered as safety factors but rather be taken into account in the form of what are known as reducing factors in order not to obscure the focus on strength. These material-specific reducing factors have values greater than one.

$$\sigma_{perm} = \underbrace{\frac{K}{S}}_{\substack{\downarrow \\ \text{not specific} \\ \text{to material}}} \cdot \underbrace{\frac{1}{A_T} \cdot \frac{1}{A_{st}} \cdot \frac{1}{A_{dyn}} \cdot \frac{1}{A_A} \cdot \frac{1}{A_W}}_{\text{material-specific}} \cdots \qquad (5.3)$$

A_T takes account of the effect of temperature on the yield stress and/or tensile strength and can be determined between 0 °C and 100 °C by the following relationship when K at 20 °C is inserted into equation (5.2).

$$A_T = \frac{1}{1 - [k\,(T - 20)]} \qquad (5.4)$$

Values for k in this equation are as follows:

PA 66   = 0.0112
PA 6   = 0.0125
PBT   = 0.0095
PA GF and PBT GF = 0.0071
POM   = 0.0082
ABS   = 0.0117

These values were obtained by linear interpolation of the plot of strength or yield stress in the temperature range between 0 and 100 °C. It is of course more sensible to safeguard against the characteristic failure value at the temperature in question, because these values are often available. On the other hand, Eq. 5.4 can also be employed to provide a rough estimate of the effect of temperature on other characteristic mechanical properties, if no experimental data are available at the required temperature. Testing does provide another option.

A_{st} takes account of the duration of a static load and can be substituted by the values of:

1.3 for a loading time of a few hours
1.6 for a loading duration measured in weeks
1.7 for a loading duration measured in months
2.0 for a loading period of a few years.

A_{dyn} takes account of the effect of dynamic loading and may be taken to be approximately 1.3 to 1.6.

A_A can also cover any aging effects (see Section 4.6.3).

Properties of materials which undergo change due to the absorption of water must be reduced by the reducing factor A_W. For unreinforced polyamides, this can be obtained starting from the value for strength in the dry state from:

$$A_W = \frac{1}{1 - 0.22\ w} \qquad (5.5)$$

where w is the moisture content in percent by weight assuming uniform distribution over the cross section. This applies within the limits $0\% < w < 3\%$. Above a moisture content of 3% by weight $A_W = 3.4$, likewise starting from strength in the dry state.

Under other conditions (*e.g.*, chemical exposure etc.), it may be necessary to consider other reducing factors.

> The process of differentiation should cause the designer to ponder deeply about any possible effects which may reduce strength. Discussions with other members of the design team can be helpful here.

5.2.2   Uniaxial State of Stress

The general equation of stress for uniaxial tensile loads having uniform distribution of stress is as follows:

$$\sigma = \frac{F}{A} \qquad (5.6)$$

Another uniaxial, but inhomogeneous, state of stress occurs in the case of bending, *e.g.*, when a beam or similar cross section has a bending moment applied at its ends.

Due to the transverse forces produced by bending loads, nonuniformly distributed shear stresses additionally appear over the cross section of the beam. These shear stresses originating from cross-force bending are, however, negligible when $l/h \geq 1$. Accordingly, pure bending may be assumed in the design of spiral springs and elastic hooks (snap-fit connections), as long as they fulfill the aforementioned condition. In practical applications, these structural parts are likewise frequently loaded beyond the linear viscoelastic range. It has, however, proved effective to use the equation for the elastic case even in such cases and in this way to determine a theoretical outer fiber stress. The values determined using elastic theory are greater than the actual stresses. When quantified, the non-linear behavior can be taken into account for a more realistic prediction.

5.2.2.1   Example of a Thin-Walled Pipe under Internal Pressure

It is extremely rare for components to be in a simple, uniaxial state of stress, although it is sometimes the case. An example is provided by a thin-walled pipe under internal pressure when only the mean tangential stress is considered. This may be regarded in approximation as a thin-walled hub mounted on a metal bolt. Under constant pressure, the stress in the interior of the pipe remains constant and the permissible maximum stress to avoid bursting of the pipe is given by:

$$\sigma_{perm} = \frac{\sigma_B\,(T, t)}{S \cdot A} \tag{5.7}$$

where σ_B for the bursting failure is taken from the stress-strain diagram or, in the case of creep failure, from the creep diagram. In Table 5.1 some results from bursting tests on thin-walled pipes are presented together with the tangential stress calculated using what is known as the "boiler formula"

$$\sigma_t = p\,\frac{r}{s} \tag{5.8}$$

using the tensile strength measured in tensile tests.

It may be seen that this comparison reveals very good agreement for a variety of different thermoplastic materials.

Table 5.1   Comparison of Calculated Burst Stress for Thin-Walled Pipes and Measured Tensile Strength Values for Several Thermoplastics

Material	Wall thickness s [mm]	Mean radius r_m [mm]	Bursting pressure p [bar]	Stress in pipe σ_t [MPa]	Tensile strength σ_R [MPa]
HDPE (5261 Z)	1.5	5.0	85	28.3	28
HDPE (5261 Z)	1.0	4.5	64	28.8	28
HDPE (5261 Z)	0.75	4.25	50	28.3	28
PVC	0.75	5.25	91	63.7	58
PVC	0.75	5.25	89	62.3	58
PMMA (G 55)	0.75	6.25	72	60.0	62
PMMA (G 55)	0.75	4.75	96	60.8	62

Example

What is the bursting pressure of a pipe made of POM GF20 at RT for which

s = 1.5 mm
r_m = 5.0 mm

and what is the approximate increase in diameter when this happens?

From CAMPUS, single-point data, Ticona, for Hostaform C 9221 GV1/20:

σ_B = 105 MPa
ε_B = 2.5%

$$p = \frac{\sigma_B \cdot s}{r_m} = \frac{105 \cdot 1.5}{5} = 31.5\,\text{N/mm}^2 = 315\,\text{bar}$$

$$\Delta d = d \cdot \varepsilon = \frac{5 \cdot 2.5}{100} = 0.125\,\text{mm}$$

The pipe ruptures at approximately 315 bar and the increase in diameter is approximately 0.125 mm.

5.2.3   Multiaxial State of Stress

Most engineering components are subjected to a multiaxial state of stress due to the external forces acting on them. The question whether this state of stress will result in failure is answered with the help of a suitable failure criterion. Using mathematical formulae for all possible states of stress resulting in failure, a multiaxial state of stress is converted to a theoretical, uniaxial resultant load, which acts on the material in a manner comparable with the actual multiaxial load and may cause failure. As a general rule, this theoretical load is taken as the characteristic tensile failure value. Failure criteria can be developed on the basis both of physical models of the failure event as well as mathematical approaches derived from empirical observations.

Because the effort involved in verifying test data experimentally is considerable, there are few confirmed results to verify that this approach can be used reliably to design polymeric material structures subjected to multiaxial stresses. References [5.3] and [5.4] provide an overview of currently known criteria.

5.2.3.1   Failure Criteria

"Classical" engineering failure criteria do not describe the behavior of polymeric materials with sufficient accuracy. Nevertheless, the shear experiments discussed in Section 5.2.3.2 show, for example, that some states of stress involving only shear may be described with sufficient apparent accuracy by Tresca's simple mathematical shear stress criterion:

$$\sigma_v = \sigma_1 - \sigma_3 \tag{5.9}$$